Nacionalni centar za vanjsko vrednovanje obrazovanja

Identifikacijska naljepnica

PAŽLJIVO NALIJEPITI

MATEMATIKA

KNJIŽICA FORMULA

viša razina

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Matematika

Knjižica formula

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F O R M U L E

- Kompleksan broj: $i^2 = -1,\ z = a + bi,\ \ \bar{z} = a - bi,\ \ |z| = \sqrt{a^2 + b^2},\ \ a, b \in \mathbf{R}$
- $z = r(\cos\varphi + i\sin\varphi)$, $z_1 \cdot z_2 = r_1 r_2 \left(\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)\right)$,

  $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)\right)$, $z^n = r^n(\cos n\varphi + i\sin n\varphi)$,

  $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\varphi + 2k\pi}{n}\right) + i\sin\left(\frac{\varphi + 2k\pi}{n}\right)\right),\ k = 0, 1, ..., n-1$

- $a^m \cdot a^n = a^{m+n}$, $a^m : a^n = a^{m-n}(a \neq 0)$, $a^{-m} = \frac{1}{a^m}(a \neq 0)$, $\sqrt[m]{a^n} = a^{\frac{n}{m}}$
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$, $\qquad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$
- $a^2 - b^2 = (a - b)(a + b)$, $\qquad a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
- $(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + ... + \binom{n}{k}a^{n-k}b^k + ... + \binom{n}{n-1}ab^{n-1} + b^n$

- Kvadratna jednadžba: $ax^2 + bx + c = 0,\ a \neq 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Vièteove formule: $x_1 + x_2 = -\frac{b}{a},\ \ x_1 \cdot x_2 = \frac{c}{a}$
- Tjeme parabole: $T\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

- $b^x = a \Leftrightarrow x = \log_b a$, $\qquad \log_b b^x = x = b^{\log_b x}$
- $\log_b(xy) = \log_b x + \log_b y$, $\log_b \frac{x}{y} = \log_b x - \log_b y$, $\log_b x^y = y\log_b x$, $\log_a x = \frac{\log_b x}{\log_b a}$

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- Površina trokuta: $P = \frac{a \cdot v_a}{2}$, $P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$, $s = \frac{a+b+c}{2}$

  $P = \frac{ab \sin\gamma}{2}$, $P = \frac{abc}{4r_o}$, $P = r_u s$

- Jednakostraničan trokut: $P = \frac{a^2\sqrt{3}}{4}$, $v = \frac{a\sqrt{3}}{2}$, $r_o = \frac{2}{3}v$, $r_u = \frac{1}{3}v$

- Površina paralelograma: $P = a \cdot v$
- Površina kruga: $P = r^2\pi$
- Površina kružnoga isječka: $P = \frac{r^2\pi\alpha}{360}$
- Površina trapeza: $P = \frac{a+c}{2} \cdot v$
- Opseg kruga: $O = 2r\pi$
- Duljina kružnoga luka: $l = \frac{r\pi\alpha}{180}$

B = površina osnovke (baze), P = površina pobočja, h = duljina visine, r = polumjer osnovke stošca

- Obujam (volumen) prizme i valjka: $V = B \cdot h$
- Obujam (volumen) piramide i stošca: $V = \frac{1}{3}B \cdot h$
- Obujam (volumen) kugle: $V = \frac{4}{3}r^3\pi$
- Oplošje prizme i valjka: $O = 2B + P$
- Oplošje piramide: $O = B + P$
- Oplošje stošca: $O = r^2\pi + r\pi s$
- Oplošje kugle: $O = 4r^2\pi$

  r = polumjer kugle

- U pravokutnome trokutu:

$$\text{sinus kuta} = \frac{\text{nasuprotna kateta}}{\text{hipotenuza}}, \quad \text{kosinus kuta} = \frac{\text{priležeća kateta}}{\text{hipotenuza}},$$

$$\text{tangens kuta} = \frac{\text{nasuprotna kateta}}{\text{priležeća kateta}}$$

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- Poučak o sinusima: $\dfrac{a}{\sin\alpha}=\dfrac{b}{\sin\beta}=\dfrac{c}{\sin\gamma}$
- Poučak o kosinusima: $c^2=a^2+b^2-2ab\cos\gamma$
- $\sin^2 x+\cos^2 x=1$, $\operatorname{tg} x=\dfrac{\sin x}{\cos x}$, $\sin 2x=2\sin x\cos x$, $\cos 2x=\cos^2 x-\sin^2 x$
- $\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$, $\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$, $\operatorname{tg}(x\pm y)=\dfrac{\operatorname{tg} x\pm\operatorname{tg} y}{1\mp\operatorname{tg} x\cdot\operatorname{tg} y}$
- $\sin x+\sin y=2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$, $\sin x-\sin y=2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

  $\cos x+\cos y=2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$, $\cos x-\cos y=-2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$
- $\sin x\sin y=\dfrac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$, $\cos x\cos y=\dfrac{1}{2}\left[\cos(x-y)+\cos(x+y)\right]$,

  $\sin x\cos y=\dfrac{1}{2}\left[\sin(x-y)+\sin(x+y)\right]$
- $\sin\dfrac{\pi}{6}=\dfrac{1}{2}$, $\sin\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$, $\sin\dfrac{\pi}{3}=\dfrac{\sqrt{3}}{2}$

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- Udaljenost točaka T_1, T_2: $d(T_1,T_2)=\sqrt{(x_2-x_1)^2+\ y_2-y_1)^2}$
- Polovište dužine $\overline{T_1T_2}$: $P\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
- Vektor $\overrightarrow{T_1T_2}$: $\overrightarrow{T_1T_2}=\vec{a}=(x_2-x_1)\vec{i}+(y_2-y_1)\vec{j}=a_1\vec{i}+a_2\vec{j}$
- Skalarni umnožak vektora: $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cdot\cos\alpha$, $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2$
- Jednadžba pravca: $y-y_1=k(x-x_1)$, $k=\frac{y_2-y_1}{x_2-x_1}$
- Kut α između dvaju pravaca: $tg\alpha=\left|\frac{k_2-k_1}{1+k_1k_2}\right|$
- Udaljenost točke $T(x_1,y_1)$ i pravca *p*... $Ax+By+C=0$: $d(T,p)=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Krivulja drugoga reda	**Jednadžba**	**Tangenta u točki krivulje (x_1, y_1)**
Kružnica središte $S(p,q)$	$(x-p)^2+(y-q)^2=r^2$	$(x_1-p)(x-p)+(y_1-q)(y-q)=r^2$
Elipsa fokusi $F_{1,2}(\pm e,0)$ $e^2=a^2-b^2$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x_1x}{a^2}+\frac{y_1y}{b^2}=1$
Hiperbola fokusi $F_{1,2}(\pm e,0)$ $e^2=a^2+b^2$ asimptote $y=\pm\frac{b}{a}x$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{x_1x}{a^2}-\frac{y_1y}{b^2}=1$
Parabola fokus $F\left(\frac{p}{2},0\right)$	$y^2=2px$	$y_1y=p(x+x_1)$

- Uvjet dodira pravca $y=kx+l$ i kružnice: $r^2(1+k^2)=(kp-q+l)^2$

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- Aritmetički niz: $a_n = a_1 + (n-1)\cdot d$, $S_n = \frac{n}{2}(a_1 + a_n)$
- Geometrijski niz: $a_n = a_1 \cdot q^{n-1}$, $S_n = a_1 \frac{q^n - 1}{q-1}$
- Geometrijski red: $S = \frac{a_1}{q-1}, |q|<1$

- Derivacija umnoška: $(f\cdot g)' = f'\cdot g + f\cdot g'$, Derivacija kvocijenta: $\left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$
- Derivacija kompozicije: $(f\circ g)'(x) = f'(g(x))\cdot g'(x)$
- Tangenta na graf funkcije f u $T(x_1, y_1)$: $y - y_1 = f'(x_1)\cdot(x - x_1)$
- Derivacije:

$c' = 0$	$(x^n)' = n\cdot x^{n-1},\ n\neq 0$	$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\mathrm{tg}\, x)' = \frac{1}{\cos^2 x}$

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